

Exponential and Logarithmic Function

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Logarithm Functions

- For $a > 0$ and $a \neq 1$,
 - ◆ $y = \log_a x$ if and only if $a^y = x$.

Graphing Logarithm Function

- The graph of the function $f(x) = \log_a x$
 - ◆ Pass through the point $(1, 0)$.
 - ◆ Has domain $(0, \infty)$ and range $(-\infty, \infty)$.
 - ◆ Approaches, but does not touch, the y -axis.
 - ◆ Increase from Left to Right if $a > 1$.
 - ◆ Decrease from Left to Right if $0 < a < 1$.

Example

- Sketch the graph of $g(x) = \log_2(x)$ and compare it to the graph $y = 2^x$.

Example

- Evaluate $\log_5(625)$

Properties of Logarithms

Product Rule for Logarithms

$$\log_a(MN) = \log_a(M) + \log_a(N)$$

Quotient Rule for Logarithms

$$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$$

Power Rule for Logarithms

$$\log_a(M^N) = N \cdot \log_a(M)$$

Summary of Properties

Properties of Logarithms

If M , N , and a are positive numbers, $a \neq 1$, then

1. $\log_a(a) = 1$
2. $\log_a(1) = 0$
3. $\log_a(a^M) = M$ Inverse properties
4. $a^{\log_a(M)} = M$
5. $\log_a(MN) = \log_a(M) + \log_a(N)$ Product rule
6. $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$ Quotient rule
7. $\log_a\left(\frac{1}{N}\right) = -\log_a(N)$
8. $\log_a(M^N) = N \cdot \log_a(M)$ Power rule

Changing the Base

Base-Change Formula

If a and b are positive numbers not equal to 1 and M is positive, then

$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

- Example:
 - ◆ Evaluate $\log_7 99$ using base-change formula

Strategy for Solving Equations

Solving Exponential and Logarithmic Equations

1. If the equation has a single logarithm or a single exponential expression, rewrite the equation using the definition $y = \log_b(x)$ if and only if $a^y = x$.
2. Use the properties of logarithms to combine logarithms as much as possible.
3. Use the one-to-one properties:
 - a) If $\log_b(m) = \log_b(n)$, then $m = n$.
 - b) If $a^m = a^n$, then $m = n$.
4. To get an approximate solution of an exponential equation, take the common or natural logarithm of each side of the equation.

Example

- Solve $3\log_8(2x - 1) = 4$.

- Solve $4^{2x} = 3^{3x-1}$.

Nonlinear System of Equations

- Equations such as
 $y = x^2$, $y = |x|$, $y = 2^x$, $y = \log_n(x)$
are nonlinear equations because their graphs are not straight line. We say that a system of equations is nonlinear if at least one equation in the system is nonlinear.

Example

- Solve the system:
 - ◆ $y = x^2 - 1$
 - ◆ $x + y = 1$

- Solve the system:
 - ◆ $x^2 + y^2 + 2y = 3$
 - ◆ $x^2 - y = 5$

Example

Solve the system:

$$(1) \quad \frac{2}{x} + \frac{1}{y} = \frac{1}{5}$$
$$(2) \quad \frac{1}{x} - \frac{3}{y} = \frac{1}{3}$$

Example

- Solve the system:
 - ◆ $y = \log_2(x + 28)$
 - ◆ $y = 3 + \log_2 x$

Example

- A 15-foot ladder is leaning against a wall so that the distance from the bottom of the ladder to the wall is one-half the distance from the top of the ladder to the ground. Find the distance from the top of ladder to the ground.

Nonlinear Inequalities

Solve $\frac{(3x+2)(3-x)}{x(x+2)(x-1)} \geq 0$.